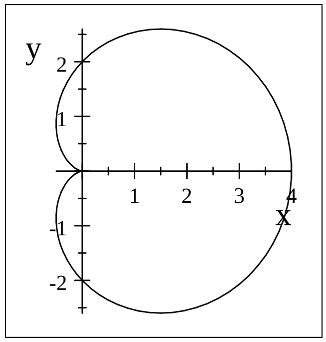
Example 1 (Titik pusat massa dalam polar) *Diberikan benda lamina dalam kardioda* $r = 2 + 2\cos\theta, 0 \leq \theta \leq 2\pi,$ *dengan rapat massa* $\rho(r,\theta) = r$. *Tentukan pusat massa benda ini.*



Solution 2 *Kita tetap menggunakan konsep momen untuk menentukan pusat massa benda ini yaitu*

$$\bar{x} = \frac{M_y}{M} = \frac{\iint\limits_D x\,dm}{\iint\limits_D dm} = \frac{\iint\limits_D (r\cos\theta)\,\rho(r,\theta)\,dA}{\iint\limits_D \rho(r,\theta)\,dA} = \frac{\iint\limits_D (r\cos\theta)\,\rho(r,\theta)\,r\,dr\,d\theta}{\iint\limits_D \rho(r,\theta)\,r\,dr\,d\theta}$$

$$\bar{y} = \frac{M_x}{M} = \frac{\iint\limits_D y\,dm}{\iint\limits_D dm} = \frac{\iint\limits_D (r\sin\theta)\,\rho(r,\theta)\,dA}{\iint\limits_D \rho(r,\theta)\,dA} = \frac{\iint\limits_D (r\sin\theta)\,\rho(r,\theta)\,r\,dr\,d\theta}{\iint\limits_D \rho(r,\theta)\,r\,dr\,d\theta}$$

Maka

$$M = \iint\limits_D \rho(r,\theta)\,r\,dr\,d\theta = \int_0^{2\pi}\int_0^{2+2\cos\theta} r \times r\,dr\,d\theta = \int_0^{2\pi}\left(\int_0^{2+2\cos\theta} r^2\,dr\right)d\theta$$
$$= \frac{1}{3}\int_0^{2\pi}(2+2\cos\theta)^3\,d\theta = \frac{40}{3}\pi$$

Dan

$$M_y = \iint\limits_D (r\cos\theta)\,\rho(r,\theta)\,r\,dr\,d\theta = \int_0^{2\pi}\int_0^{2+2\cos\theta}(r\cos\theta)\,r \times r\,dr\,d\theta$$
$$= \int_0^{2\pi}\left(\int_0^{2+2\cos\theta}\left(r^3\cos\theta\right)dr\right)d\theta = \int_0^{2\pi}\cos\theta\left(\int_0^{2+2\cos\theta} r^3\,dr\right)d\theta$$
$$= \int_0^{2\pi}\cos\theta\left(4(\cos\theta+1)^4\right)d\theta = 4\int_0^{2\pi}\cos\theta(\cos\theta+1)^4\;d\theta = 28\pi$$

$$
\begin{aligned}
M_x &= \iint\limits_D (r\sin\theta)\,\rho(r,\theta)\,r\,dr\,d\theta = \int_0^{2\pi}\int_0^{2+2\cos\theta}(r\sin\theta)\,r\times r\,dr\,d\theta \\
&= \int_0^{2\pi}\left(\int_0^{2+2\cos\theta}\left(r^3\sin\theta\right)dr\right)d\theta = \int_0^{2\pi}\sin\theta\left(\int_0^{2+2\cos\theta}r^3dr\right)d\theta \\
&= \int_0^{2\pi}\sin\theta\left(4\left(\cos\theta+1\right)^4\right)d\theta = 0.
\end{aligned}
$$

Maka,

$$
\begin{aligned}
\bar{x} &= \frac{28\pi}{\frac{40}{3}\pi} = \frac{21}{10} \\
\bar{y} &= \frac{0}{\frac{40}{3}\pi} = 0
\end{aligned}
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